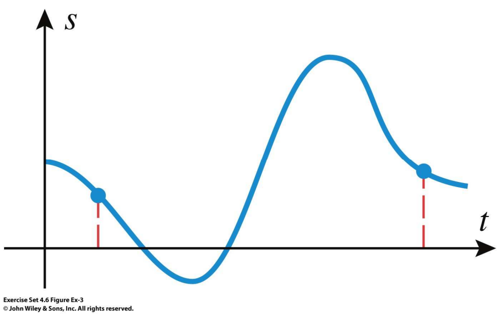
Mean-Value Theorem

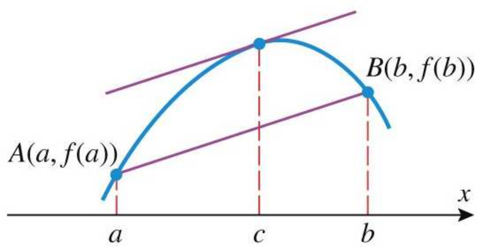
Warm-up You are driving on a straight highway on which the speed limit is 55 mph. At 8:05 am a police car clocks your velocity at 50 mph and at 8:10 am a second police car posted 5 miles down the road clocks your velocity at 55 mph. Does the police officer have a right to charge you with a speeding violation?

Consider the following graph to understand the Mean-Value Theorem:



> ***Mean-Value Theorem: Let f be continuous on the closed interval [a, b] and differentiable on the open interval (a, b). Then there is at least one point c in (a, b) such that***
>
> $$f''(c) = \frac{f(b) - f(a)}{b - a}$$

Illustration of the Mean-Value Theorem



Example 1 Given $f(x) = \frac{1}{4}x^3 + 1$. Find all values of c in the interval (0, 2) guaranteed by the Mean-Value Theorem.

Practice Problem 1 Let $f(x) = x^2 - x$. Find all values of c in the interval (0, 2) guaranteed by the Mean-Value Theorem.

Mean-Value Theorem

Velocity Interpretation of the Mean-Value Theorem

Apply the Mean-Value Theorem to the position versus time graph:

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

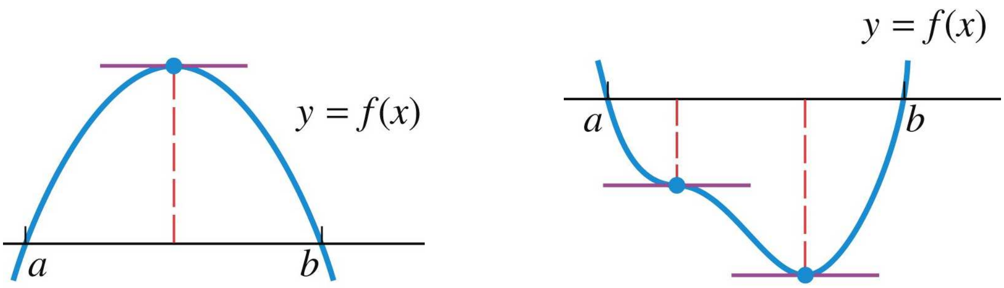
The right side of this formula would be ____.

The left side of this formula would be ____.

Thus, the Mean-Value Theorem implies that at least once during the time interval, the instantaneous velocity must equal the average velocity. This agrees with our real-world experience – if the average velocity for our trip is 40 mph, then sometime during our trip the speedometer has to read 40 mph (even if only momentarily).

Example 2 Calculate your average velocity from the warm-up problem. Explain how this indicates that the police officer does have the right to give you a speeding ticket.

Rolle's Theorem – A Special Case of the Mean-Value Theorem



Rolle's Theorem: Let f be continuous on the closed interval [a, b] and differentiable on the open interval (a, b). If

f(a) = 0 and f(b) = 0

then there is at least one point c in the interval (a, b) such that f'(c) = 0.

Example 3 Find the two *x*-intercepts of the function $f(x) = x^2 - 5x + 4$, and then find the value *c* that is guaranteed by Rolle's Theorem.

Mean-Value Theorem

Class Work

Find all values of *c* in the interval that are guaranteed by Rolle's Theorem.

1. $f(x) = x^2 - 8x + 15 \quad [3, 5]$

2. $f(x) = \frac{1}{2}x - \sqrt{x} \quad [0, 4]$

3. $f(x) = \cos x \quad \left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$

4. $f(x) = \ln(4 + 2x - x^2) \quad [-1, 3]$

Find all values of *c* in the interval that are guaranteed by the Mean-Value Theorem.

5. $f(x) = x^3 - 4x \quad [-2, 1]$

6. $f(x) = x^3 + x - 4 \quad [-1, 2]$

7. $f(x) = \sqrt{25 - x^2} \quad [-5, 3]$

8. $f(x) = x - \frac{1}{x} \quad [3, 4]$